SOLUTIONS & ANSWERS FOR KERALA ENGINEERING ENTRANCE EXAMINATION-2010 – PAPER II VERSION – B1

[MATHEMATICS]

1. Ans: $1+\frac{1}{a}$

Sol: $\frac{\alpha a^2}{a+1}=a$

$\Rightarrow \alpha = 1 + \frac{1}{a}$.

2. Ans: –4

Sol: $a * b = 5$

$\Rightarrow \frac{a}{a+b}=5$

$a * b + b * a = \frac{a}{a+b}+\frac{b}{a+b}=1$

$\Rightarrow b * a = 1 - a * b = 1 - 5 = -4$.

3. Ans: {(z, b), (y, b), (a, d)}

Sol: Conceptual. The first element of (a, d) is not from A.

4. Ans: [6, 24]

Sol: $-1 \le \log_2\left(\frac{x}{12}\right) \le 1$

$\Rightarrow \frac{1}{2} \le \frac{x}{12} \le 2$

$\Rightarrow 6 \le x \le 24$

$\Rightarrow$ domain = [6, 24]

5. Ans: x^4

Sol: $(g \circ f)(x) = [x^2 - 1 + 1]^2$

$= x^4$.

6. Ans: $(A - B) \cup (B - A)$.

Sol: Conceptual. It is clearly $(A - B) \cup (B - A)$.

7. Ans: –120.

Sol: $x + iy = (2 + 3i)^3$

$= -46 + 9i$

$\Rightarrow x = -46, y = 9$

$\Rightarrow 3x + 2y = -120$.

8. Ans: 3

Sol: $z = x + iy$

$\Rightarrow (x + 3)^2 + (y - 1)^2 = 1$ and

$\frac{y}{x}=0$

$\Rightarrow Y = 0, X = -3$

$|z| = \sqrt{9+0}=3$.

9. Ans: $\left|\frac{1}{z_1}+\frac{1}{z_2}+\ldots\frac{1}{z_n}\right|$

Sol: $\left|z_1+z_2+\ldots\ldots+z_n\right| = \left|\bar{z}_1+\bar{z}_2+\ldots+\bar{z}_n\right|$

10. Ans: i

Sol: $\frac{\left(\sqrt{3}+i\right)\left(1+i\sqrt{3}\right)}{\left(1-i\sqrt{3}\right)\left(1+i\sqrt{3}\right)}=i.$

11. Ans: $2\cos 2\theta$.

Sol: $(\cos\theta + i\sin\theta)^2 + (\cos\theta - i\sin\theta)^2$

$= 2\cos 2\theta$.

12. Ans: $\sqrt{6}$

Sol: $\left|z_1 z_2\right| = \left|z_1\right|\left|z_2\right|$.

13. Ans: a = 3.

Sol: $\left(2\sqrt{6}\right)^2 - 8a = 0$

14. Ans: a + b = 31

Sol: $\left(x-\left(\frac{3}{2}+\frac{7i}{2}\right)\right)\left(x-\left(\frac{3}{2}-\frac{7i}{2}\right)\right) = 0$

$\Rightarrow 2x^2 - 6x + 29 = 0$

$\Rightarrow a + b = 31$

15. Ans: $b^2 - 4c = 1$

Sol: If α and $\alpha + 1$ are the roots, $2\alpha + 1 = b$ and $\alpha(\alpha + 1) = c$

$\left(\frac{b-1}{2}\right)\left(\frac{b+1}{2}\right)=c$

$\Rightarrow b^2 - 4c = 1$.

16. Ans: $acx^2 - bx + 1 = 0$

Sol: $\alpha + \beta = -\frac{b}{a},\ \alpha\beta=\frac{c}{a}$

Sum $= \frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$

$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$= \frac{b}{ac}$

Product $= \frac{1}{ac}$

$x^2 - \frac{b}{ac}x + \frac{1}{ac} = 0$

$\Rightarrow acx^2 - bx + 1 = 0$.

17. Ans: 1 and –2

Sol: Let a and b be the roots
$\Rightarrow a + b = -a$
$ab = b$
$\Rightarrow a = 1$ and $b = -2$ (from options)

18. Ans: $p^2 - 4q$

Sol: $\alpha + \beta = -p$; $\alpha\beta = q$
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$= p^2 - 4q$.

19. Ans: 15

Sol: Given, $n(n + 1) = 240 = 15 \times 16$
comparing; $n = 15$.

20. Ans: –8

Sol: sum $= \left(\sqrt{10}+\sqrt{9}\right)-\left(\sqrt{11}+\sqrt{10}\right)+\left(\sqrt{12}+\sqrt{11}\right) + \ldots + \left(\sqrt{120}+\sqrt{119}\right)-\left(\sqrt{121}-\sqrt{120}\right)$

$= \sqrt{9}-\sqrt{121} = 3 - 11 = -8$.

21. Ans: 3

Sol: $a_{11} + a_{12} + a_{13} = 141$
and $a_{21} + a_{22} + a_{23} = 261$
$\Rightarrow 3a + 33d = 141$ and $3a + 63d = 261$
$\Rightarrow a = 3$.

22. Ans: $\tan^{-1}\left(\frac{5n-5}{1+a_n a_1}\right)$

Sol: $\tan^{-1}\left[\frac{a_2-a_1}{1+a_1a_2}\right]+\tan^{-1}\left[\frac{a_3-a_2}{1+a_2a_3}\right]+ \ldots + \tan^{-1}\left[\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right]$

$= \tan^{-1}a_2 - \tan^{-1}a_1 + \ldots + \tan^{-1}a_n - \tan^{-1}a_{n-1}$

$= \tan^{-1}a_n - \tan^{-1}a_1$

$= \tan^{-1}\frac{a_n-a_1}{1+a_1a_n} = \tan^{-1}\left(\frac{5n-5}{1+a_na_1}\right)$

23. Ans: 702.

Sol: $12 + 19 + \ldots + 96$
$= \frac{13}{2}[12+96] = 702$.

24. Ans: 8.

Sol: $\frac{a+2}{2}-\sqrt{2a} = 1$
$\Rightarrow \frac{a+2}{2} - 1 = \sqrt{2a}$
$\Rightarrow a^2 = 8a$
$\Rightarrow a = 8$.

25. Ans: $8.\ ({}^6C_3)$

Sol: The middle term is the fourth term.
$T_4 = {}^6C_3 \,.\, x^3 \,.\, (2y)^3$
$= 8.\ ({}^6C_3) \,.\, x^3 y^3$
$\therefore$ coefficient $= 8.\ ({}^6C_3)$

26. Ans: 7

Sol: Given ${}^nC_1$, ${}^nC_2$, ${}^nC_3$ are in A.P
$\Rightarrow 2.\frac{n(n-1)}{2} = n + \frac{n(n-1)(n-2)}{6}$
$\Rightarrow n^2 - 9n + 14 = 0 \Rightarrow (n - 1)(n - 2) = 0$
$\Rightarrow n = 7$ from options.

27. Ans: 72

Sol: There are 5 places, out of which the first can be filled in 3 ways. The remaining four places are filled in 4! = 24 ways
$\therefore$ Total number of integers $= 3 \times 24 = 72$.

28. Ans: 3

Sol: $(a^2 - 6a + 11)^{10} = 2^{10}$
$\Rightarrow a^2 - 6a + 11 = 2$
$6a + 11 = 2$
$\Rightarrow a^2 - 6a + 9 = 0$
$\Rightarrow a = 3$, or $a = 3$.

29. Ans: 41

Sol: ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$
$\Rightarrow \frac{55\times56}{(50-r)!}\times\frac{(51-r)!}{54!} = \frac{30800}{1}$
$\Rightarrow r = 41$

30. Ans: 132

Sol: Required difference $= {}^{11}C_5 - {}^{11}C_4$
$= 462 - 330$
$= 132$.

31. Ans: 4

Sol: $A^{-1} = \frac{1}{7x+6}\begin{bmatrix} 7 & 2 \\ -3 & x \end{bmatrix}$
$\Rightarrow 7x + 6 = 34$
$\Rightarrow x = 4$.

32. Ans: 21.

Sol: To get a_0, put $x = 0$

$$a_0 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix} = 21.$$

33. Ans: 2(15!) (16!) (17!)

Sol: 2n! (n + 1)! (n + 2)!
= 2(15!) (16!) (17!).

34. Ans: |A|A

Sol: $\text{adj(adj A)} = |A|^{n-2}A$
$= |A|A.$

35. Ans: x = 1, y = –2, z = 3.

Sol: x – y – z = 0, –y + z = 5 and z = 3
$\Rightarrow$ y = –2 and x = 1.

36. Ans: $B^{-1}A^{-1}$.

Sol: $(AB)^{-1} = B^{-1}A^{-1}$.

37. Ans: $(-\infty, -11) \cup (3, \infty)$

Sol: x + 11 > 0 and x – 3 > 0
or x + 11 < 0 and x – 3 < 0
$\Rightarrow$ x > –11 and x > 3
or x < –11 and x < 3 $\Rightarrow$ x > 3 or x < –11
$\Rightarrow (-\infty, -11) \cup (3, \infty)$.

38. Ans: $8 \le t + 1 \le 13$

Sol: $21 \le 3t \le 36$
$\Rightarrow 7 \le t \le 12$
$\Rightarrow 8 \le t + 1 \le 13.$

39. Ans: 7 is greater than 4 and Paris is not in France

Sol: $\sim(p \vee q) \equiv \sim p \wedge \sim q$
$\Rightarrow$ 7 is greater than 4 and Paris is not in France.

40. Ans: $p \vee (q \vee r)$

Sol: $S(p, q, r) \equiv \sim[p \wedge q \wedge r]$
$S(\sim p, \sim q, \sim r) \equiv \sim[\sim p \wedge \sim q \wedge \sim r]$
$\equiv [p \vee (q \vee r)]$

41. Ans: ~p.

Sol: Using truth table, the given expression is logically equivalent to ~p.

42. Ans: $\dfrac{2\sin\alpha}{\sqrt{\cos 2\alpha}}$

Sol: $\sqrt{\dfrac{a+b}{a-b}} - \sqrt{\dfrac{a-b}{a+b}} = \dfrac{2b}{\sqrt{a^2-b^2}}$

$$= \frac{2\frac{b}{a}}{\sqrt{1-\left(\frac{b}{a}\right)^2}} = \frac{2\tan\alpha}{\sqrt{1-\tan^2\alpha}}$$

$$= \frac{2\sin\alpha}{\sqrt{\cos^2\alpha - \sin^2\alpha}} = \frac{2\sin\alpha}{\sqrt{\cos 2\alpha}}.$$

43. Ans: x = 1

Sol: $\tan^{-1}\left(\dfrac{x+2+x-2}{1-(x^2-4)}\right) = \tan^{-1}\dfrac{1}{2}$,

(x + 2) (x – 2) < 1

$\dfrac{2x}{5-x^2} = \dfrac{1}{2},\ x^2 < 5$
$\Rightarrow x^2 + 4x - 5 = 0$
$\Rightarrow$ x = –5, 1

44. Ans: $\dfrac{63}{65}$

Sol: $\sin\alpha = \dfrac{4}{5} \Rightarrow \cos\alpha = \dfrac{3}{5}$

$\cos(\alpha+\beta) = \dfrac{-12}{13} \Rightarrow \sin(\alpha+\beta) = \dfrac{5}{13}$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta = \dfrac{-12}{13}$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta = \dfrac{5}{13}$

substituting the values and solving,

$\sin\beta = \dfrac{63}{65}$.

45. Ans: Number of solutions in $(0, 2\pi)$ is 3.

Sol: $1 - 2\sin^2\theta = \sin\theta$
$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$
$\Rightarrow \sin\theta = -1, \dfrac{1}{2}$
$\Rightarrow \theta = \dfrac{3\pi}{2}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$.

46. Ans: $\dfrac{\pi}{2}$

Sol: $\sin^{-1}\dfrac{4}{5} + 2\tan^{-1}\dfrac{1}{3}$

$= \tan^{-1}\dfrac{4}{3} + \tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{3}$

$= \tan^{-1}3 + \tan^{-1}\dfrac{1}{3}$

$= \tan^{-1}\dfrac{3+\frac{1}{3}}{1-1} = \dfrac{\pi}{2}$.

47. Ans: $\sqrt{3}$

Sol: $\tan 60 = \sqrt{3} = \tan(40+20)$

$= \dfrac{\tan 40 + \tan 20}{1 - \tan 40 \tan 20}$

$\Rightarrow \tan 40 + \tan 20 + \sqrt{3}\tan 40 \tan 20 = \sqrt{3}$.

48. Ans: $\dfrac{2\pi}{3}$

Sol: $f(\theta) = 4 - \sin 3\theta$

period of $\sin\theta = 2\pi$

$\Rightarrow$ period of $\sin 3\theta = \dfrac{2\pi}{3}$.

49. Ans: $\dfrac{\pi}{12}$

Sol: $\sin x \cos x = \dfrac{1}{4} \Rightarrow \sin 2x = \dfrac{1}{2} = \sin\dfrac{\pi}{6}$

$\Rightarrow 2x = \dfrac{\pi}{6} \Rightarrow x = \dfrac{\pi}{12}$.

50. Ans: $-\dfrac{\pi}{4}$

Sol: $\cos 4095° = \cos 135° = -\dfrac{1}{\sqrt{2}}$

$\therefore \sin^{-1}\left(-\dfrac{1}{\sqrt{2}}\right) = -\dfrac{\pi}{4}$.

51. Ans: –6, 8

Sol: $(x-1)^2 + (2-3)^2 = (2+5)^2 + (3-2)^2$

$\Rightarrow (x-1)^2 = 49$

$\Rightarrow x - 1 = \pm 7$

$\Rightarrow x = 8, -6$.

52. Ans: 8

Sol: The circle is $(x-4)(x+2) + (y-7)(y+1) = 0$

$\Rightarrow x^2 + y^2 - 2x - 6y - 15 = 0$

The points A and B are given by putting $y = 0$

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow x = -3, 5$

$\Rightarrow AB = 8$.

53. Ans: $\sqrt{3}, -\sqrt{3}$

Sol: $4 = x^2 + 1$

$\Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$.

54. Ans: $\dfrac{3}{2}$

Sol: Area $= \dfrac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ 5 & 5 & 1 \\ 6 & 7 & 1 \end{vmatrix} = \dfrac{3}{2}$ sq. units.

55. Ans: $\dfrac{r(q-p)}{pq}$

Sol: Substituting (a, 0) and (0, b) in the line,

$a = \dfrac{r}{p}$; $b = \dfrac{-r}{q}$

$\therefore a + b = \dfrac{r}{p} - \dfrac{r}{q} = \dfrac{r(q-p)}{pq}$.

56. Ans: $y = x + 1$

Sol: Δ ABC is right angled at B and BC is parallel to the x-axis. Also, bisector must pass through B.
So only option which satisfies these conditions is $y = x + 1$.

57. Ans: $x\cos\theta - y\sin\theta = a\cos 2\theta$.

Sol: $\dfrac{x}{\cos\theta} + \dfrac{y}{\sin\theta} = a$

$\Rightarrow x\sin\theta + y\cos\theta = a\sin\theta\cos\theta$

$\Rightarrow$ required line is $x\cos\theta - y\sin\theta = k$

$\Rightarrow$ since it passes through $(a\cos^3\theta, a\sin^3\theta)$,

$k = a\cos 2\theta \Rightarrow x\cos\theta - y\sin\theta = a\cos 2\theta$.

58. Ans: -2 or $\dfrac{1}{2}$.

Sol: Slope of $3x - y = -5$ is $m_1 = 3$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right| = 1$

$\Rightarrow \dfrac{m_2 - 3}{1 + 3m_2} = \pm 1$

$\Rightarrow m_2 = -2$ or $\dfrac{1}{2}$.

59. Ans: $4x - 3y - 3 = 0$

Sol: Parallel to given lines only possible options are (C) or (D) or (E).
Distance from (–1, –4) = 1 unit is satisfied by $4x - 3y - 3 = 0$.

60. Ans: $x^2 + y^2 - 2hx - 2ky + h^2 = 0$

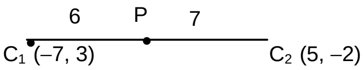
Sol: radius = k, centre = (h, k)

equation is $(x-h)^2 + (y-k)^2 = k^2$

$\Rightarrow x^2 + y^2 - 2hx - 2ky + h^2 = 0$.

61. Ans: $P\left(\frac{-19}{13}, \frac{9}{13}\right)$

Sol:



$P\left(\frac{-19}{13}, \frac{9}{13}\right)$

62. Ans: $2x - 3y = -13$

Sol: $xx_1 + yy_1 = x_1^2 + y_1^2$
$-2x + 3y = (-2)^2 + 3^2$
$2x - 3y = -13$

63. Ans: $2\sqrt{2}$

Sol: Mid point of segment (4, 0) and (0, 4) is (2, 2). Distance from (0, 0) to (2, 2) = $2\sqrt{2}$

64. Ans: $\left(9, 6\sqrt{3}\right)$

Sol: $y^2 = 12x$
focus (3, 0)
focal distance = 12
Point $\left(9, 6\sqrt{3}\right)$

65. Ans: $\frac{15}{17}$

Sol: $2a = \frac{17}{8} \times 2b \Rightarrow b = \frac{8}{17}a$

$e^2 = \frac{a^2 - b^2}{a^2} = \frac{a^2 - \frac{64}{289}a^2}{a^2}$

$e = \frac{15}{17}$

66. Ans: 9

Sol: Maximum CP = 5
Minimum CP = 4
Total = 9

67. Ans: 8

Sol: $7x^2 - 9y^2 = 63$

$\frac{x^2}{9} - \frac{y^2}{7} = 1$, $a = 3$

Eccentricity = $\frac{4}{3}$

Foci's are (4, 0), (–4, 0).
Distance = 8

68. Ans: $5\sqrt{11}$

Sol: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\cos\theta = \frac{-5}{6}$

$\left|\vec{a} \times \vec{b}\right|^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 (1 - \cos^2\theta)$

$= 25 \times 36 \left(1 - \frac{25}{36}\right)$

$= 25 \times 11$

$\left|\vec{a} \times \vec{b}\right| = 5\sqrt{11}$

69. Ans: 5

Sol: $\left|(\bar{p} + \bar{q}) + (\bar{q} + \bar{r}) + (\bar{r} + \bar{p})\right|^2 = 100$

$\left|\bar{p} + \bar{q} + \bar{r}\right| = \frac{10}{2} = 5$

70. Ans: 5a

Sol: Let the sides of cube be along the axes.
Diagonals have D.C's $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$, $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ and $\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore$ Adding; resultant = $a.\sqrt{9 + \frac{16}{2} + \frac{28}{2}}$

$= 5a$

71. Ans: $2\left(2\hat{i} - \hat{j} - 2\hat{k}\right)$

Sol: By back substitution.

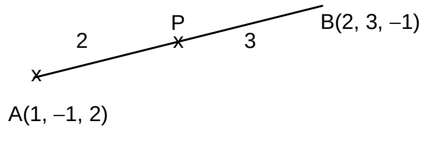
72. Ans: 1

Sol: $\begin{vmatrix} 2 & 1 & 4 \\ 4 & -2 & 3 \\ 2 & -3 & -\lambda \end{vmatrix} = 0$

$2(2\lambda + 9) - 1(-4\lambda - 6) + 4(-12 + 4) = 0$
$4\lambda + 18 + 4\lambda + 6 - 32 = 0$
$8\lambda - 8 = 0;\ \lambda = 1$

73. Ans: $\frac{1}{5}\left(7\hat{i} + 3\hat{j} + 4\hat{k}\right)$

Sol:



$p\left(\frac{7}{5}, \frac{3}{5}, \frac{4}{5}\right)$

74. Ans: 6

Sol: $\left(\hat{i}+\hat{j}+2\hat{k}\right).\left(\dfrac{m\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{m^2+13}}\right)=2$

$\dfrac{m+8}{\sqrt{m^2+13}}=2$

$\Rightarrow 3m^2 - 16\,m - 12 = 0$

$\Rightarrow m = \dfrac{-2}{3}$, $m = 6$

75. Ans: $2x + 3y + 6z = 18$

Sol: $\dfrac{a}{3}=3,\ \dfrac{b}{3}=2,\ \dfrac{c}{3}=1$

$a = 9,\ b = 6,\ c = 3$

$\dfrac{x}{9}+\dfrac{y}{6}+\dfrac{z}{3}=1$

$2x + 3y + 6z = 18$

76. Ans: $\dfrac{3}{2}$

Sol: $\dfrac{\left(x-\dfrac{1}{3}\right)}{\dfrac{2b}{3}}=\dfrac{y-3}{-1}=\dfrac{z-1}{a}$

$3\times\dfrac{2b}{3}+1\times -1+2a=6$

$2b + 2a = 1$

$a + b = \dfrac{1}{2}$

$3a + 3b = \dfrac{3}{2}$

77. Ans: $\dfrac{x-3}{2}=\dfrac{-y}{3}=\dfrac{z+4}{5}$

Sol: $\dfrac{x-3}{2}=\dfrac{y-0}{-3}=\dfrac{z+4}{5}$

78. Ans: The line parallel is

$\hat{r}=\left(-\hat{i}+\hat{j}-\hat{k}\right)+t\left(-\hat{i}-2\hat{j}+4\hat{k}\right)$

Sol: The given plane is

$\hat{r}=2\hat{i}-7\hat{j}-3\hat{k}+s\left(\hat{i}+2\hat{j}-4\hat{k}\right)+t\left(\hat{i}+2\hat{j}-4\hat{k}\right)$

Vector along normal to plane

$= 42\hat{i}-3\hat{j}+9\hat{k}$

79. Ans: 2

Sol: Point on the line (2, 2, –1)

$\therefore$ Distance = $\left|\dfrac{2+4-2-10}{\sqrt{1+4+4}}\right|$

$= \dfrac{6}{3} = 2$

80. Ans: $20x + 23y + 26z = 69$

Sol: $x + y + z - 6 + \lambda(2x + 3y + 4z + 5) = 0$

$14\lambda = 3$; $\lambda = \dfrac{3}{14}$

Equation of the plane is

$\left(1+\dfrac{6}{14}\right)x+\left(1+\dfrac{9}{14}\right)y+\left(1+\dfrac{12}{14}\right)z$

$-6 + \dfrac{15}{14} = 0$

81. Ans: $8x + y - 5z = 7$

Sol: Points (1, –1, 0) and (0, 2, –1) in the plane

$a(x - 1) + b(y + 1) + c.2 = 0$

$-a+3b-c=0$

$2a-b+3c=0$

$\dfrac{a}{8}=\dfrac{b}{1}=\dfrac{c}{-5}$

Equation is $8x + y - 5z = 7$

82. Ans: $\vec{r}=\left(\hat{i}-\hat{j}+3\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}-\hat{k}\right)$

Sol: $\dfrac{x-1}{3}=\dfrac{y--1}{2}=\dfrac{z-3}{-1}$

83. Ans: m + 4

Sol: Given 7a + 21 = 7 m

a + 3 = m $\Rightarrow$ a = m – 3

$\Rightarrow$ Required A.M = m + 4

84. Ans: $\dfrac{1}{72}$

Sol: $\Sigma pi = 1$

$72p = 1$

$p = \dfrac{1}{72}$

85. Ans: 16

Sol: Mean = 5 Variance = 0

$\sigma^2 = \dfrac{1}{n}\Sigma x_i^{\,2}-\left(\dfrac{\Sigma x_i}{n}\right)^2$

$0 = \dfrac{1}{n}\times 400-5^2 \Rightarrow 400 = 25\,n$

$n = 16$

86. Ans: $\dfrac{2}{7}$

Sol: $P(A \cup B) = \dfrac{13}{21}$ $\quad P(B) = \dfrac{1}{3}$

$P(A) + P(B) = \dfrac{13}{21}$

$P(A) = \dfrac{2}{7}$

87. Ans: 500

Sol: $f(x + y) = f(x) + f(y)$

$f(2) = f(1) + f(1) = 10$

$f(3) = f(2) + f(1) = 15$

etc.

$f(100) = 5 + 99 \times 5$

$= 5 \times 100 = 500$

88. Ans: $a = 3$

Sol: $$\underset{x\to 0}{\text{Lt}} \frac{\left(e^x - 1\right)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{4}\right)}$$

$$\underset{x\to 0}{\text{Lt}} \frac{\frac{\left(e^x-1\right)^2}{x}.x^2}{\frac{x}{a}.\frac{\sin\left(\frac{x}{a}\right)}{\left(\frac{x}{a}\right)}.\frac{\log\left(1+\frac{x}{4}\right)}{\frac{x}{4}}.\frac{x}{4}}$$

$\Rightarrow 4a = 12$

$a = 3$

89. Ans: 1

Sol: $$\underset{x\to 0}{\text{Lt}} \frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{(1+x)-(1-x)} = 1$$

90. Ans: $\frac{2}{9}$

Sol: $$\underset{x\to\infty}{\text{Lt}} \frac{3x^4+2x^3-3x^4+4x^2}{\left(3x^2-4\right)(3x+2)}$$

$$= \underset{x\to\infty}{\text{Lt}} \frac{2x^3+4x^2}{\left(3x^2-4\right)(3x+2)}$$

$$= \underset{x\to\infty}{\text{Lt}} \frac{x^3\left(2+\frac{4}{x}\right)}{x^3\left(3-\frac{4}{x^2}\right)\left(3+\frac{2}{x}\right)}$$

$$= \frac{2}{9}$$

91. Ans: $\frac{2(\log x+1)}{(\log x+2)^2}$

Sol: $x^y = e^{2(x-y)}$

$y \log x = 2(x - y)$

$y(\log x + 2) = 2x \rightarrow y = \frac{2x}{\log x + 2}$

$$\frac{dy}{dx} = \frac{2(\log x+1)}{(\log x+2)^2}$$

92. Ans: $\frac{-1}{2\sqrt{x}\sqrt{1-x}}$

Sol: $\sin y = \sqrt{1-x}$

$$\cos y\,\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x}}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}\sqrt{1-x}}$$

93. Ans: $\frac{2}{3}$

Sol: $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$; $v = \sin^{-1}(3x - 4x^3)$

$x = \sin A$ $\qquad \frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}}$

$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}}$$

$$\frac{du}{dv} = \frac{2}{3}$$

94. Ans: 0

Sol: $y = \tan^{-1}x + \sec^{-1}x + \cot^{-1}x + \text{cosec}^{-1}x$

$$\frac{dy}{dx} = 0$$

95. Ans: -3

Sol: $f(x) = |x - 2| + |x + 1| - x$

$x < -1$ $\quad f(x) = -(n - 2) - (n + 1) - x$

$= -3x + 1$

$f'(-10) = -3$

96. Ans: $\frac{-1}{a}$

Sol: $x = a(1 + \cos\theta)$ $\quad y = a(\theta + \sin\theta)$

$\frac{dx}{d\theta} = -a\sin\theta$ $\qquad \frac{dy}{d\theta} = a(1 + \cos\theta)$

$$\frac{dy}{dx} = \frac{-\cot\theta}{2}$$

$$\frac{d^2y}{dx^2} = +\frac{\text{cosec}^2\theta}{2}\times\frac{1}{2}\times\frac{-1}{a\sin\theta}$$

$\theta = \frac{\pi}{2}$ $\qquad \frac{d^2y}{dx^2} = \frac{-1}{a}$

97. Ans: $-\frac{1}{2}$

Sol: $$y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$$

$$= \tan^{-1}\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}$$

$$= \tan^{-1}\frac{2\sin\left(\frac{\pi}{2}-\frac{x}{2}\right)\cos\left(\frac{\pi}{2}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{2}-\frac{x}{2}\right)}$$

$\tan^{-1}\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)=\frac{\pi}{2}-\frac{x}{2}$

$\frac{dy}{dx}=-1$

98. Ans: $\frac{2}{\sqrt{5}}$

Sol: $y = e^{2x} + x^2$, $y' = 2e^{2x} + 2x$
y' at x = 0 is equal to 2

$\therefore$ slope of normal $= -\frac{1}{2}$

$\therefore y - 1 = -\frac{1}{2}(x) \Rightarrow 2y - 2 = -x$

$x + 2y - 2 = 0$

$\therefore$ distance $= \frac{2}{\sqrt{5}}$

99. Ans: $\frac{4}{3}$

Sol: $f(x) = x(x-1)^2 = x^3 - 2x^2 + x$
$f'(x) = 3x^2 - 4x + 1$

$f'(c) = \frac{f(b)-f(a)}{b-a}$

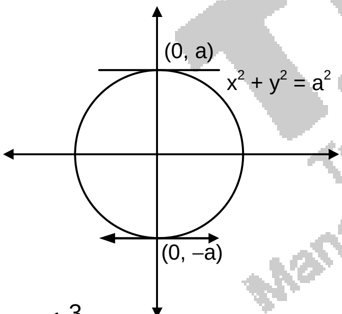
$3c^2 - 4c + 1 = \frac{f(2)-f(0)}{2-0}$

$= \frac{2-0}{2-0} = 1$

$3c^2 - 4c = 0 \Rightarrow c = \frac{4}{3}$

100. Ans:

Sol:



101. Ans: $\tan^{-1}\frac{3}{4}$

Sol: $y = x^2$ $\quad y^2 - x = 0$

$y' = 2x \quad 2yy' - 1 = 0,\ y' = \frac{1}{2y}$

At (1, 1)

$y' = 2$ and $y' = \frac{1}{2}$

$\tan\theta = \left|\frac{2-\frac{1}{2}}{1+1}\right| = \frac{3}{4} \quad \theta = \tan^{-1}\frac{3}{4}$

102. Ans: 750 cm^3/sec.

Sol: $\frac{da}{dt} = 10$ cm/sec
$v = a^3$
$\frac{dv}{dt} = 3a^2\frac{da}{dt} = 3\times5^2\times10$
$= 750\ cm^3$/sec.

103. Ans: 7

Sol: f(x) = |3 – x| + 7
Vertex = (3, 7)
Minimum value 7.

104. Ans: 0.1%

Sol: $A = \pi r^2$
$\log A = \log\pi + 2\log r$
$\frac{\Delta A}{A}\times100 = 2\frac{\Delta r}{r}\times100$
$= 2 \times 0.05$
$= 0.1\%$.

105. Ans: $\frac{1}{4}$

Sol: $x + 2 = A(4x + 6) + B$

$4A = 1 \rightarrow A = \frac{1}{4}$

$\therefore P = \frac{1}{4}$.

106. Ans: $\frac{(x+2)^{10}}{10}-\frac{(x+2)^8}{8}+C$

Sol: $\int(x+1)(x+2)^7(x+3)dx$

$= \int\left[(x+2)^2-1\right][x+2]^7dx$

$= \int\left[(x+2)^9-(x+2)^7\right]dx$

$= \frac{(x+2)^{10}}{10}-\frac{(x+2)^8}{8}+C$.

107. Ans: $2\left[\frac{(x+1)^{\frac{7}{2}}}{7}-2\frac{(x+1)^{\frac{5}{2}}}{5}+2\frac{(x+1)^{\frac{3}{2}}}{3}\right]+C.$

Sol: $\int(x^2+1)\sqrt{x+1}\,dx$
$x + 1 = t^2 \quad dx = 2t\,dt$
$x = t^2 - 1 \quad x^2 + 1 = t^4 - 2t^2 + 2$

$2\int(t^6-2t^4+2t^2)dt$

$= 2\left[\frac{t^7}{7}-2\frac{t^5}{5}+2\frac{t^3}{3}\right]$

$$= 2\left[\frac{(x+1)^{\frac{7}{2}}}{7} - 2\frac{(x+1)^{\frac{5}{2}}}{5} + 2\frac{(x+1)^{\frac{3}{2}}}{3}\right] + C.$$

108. Ans: $\log\left|1+xe^x\right| + C.$

Sol: $\int \frac{1+x}{x+e^{-x}}dx = \int \frac{xe^x+e^x}{xe^x+1}dx$

$= \log\left|1+xe^x\right| + C.$

109. Ans: $\frac{1}{2}\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2 + C$

Sol: Put $\log\left(x+\sqrt{1+x^2}\right) = t$

$dt = \frac{dx}{\sqrt{1+x^2}}$

$\int t\,dt = \frac{t^2}{2} + C = \frac{1}{2}\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2 + C.$

110. Ans: $-\log\left[e^{-x} + \sqrt{e^{-2x}-1}\right] + C$

Sol: $\int \frac{dx}{\sqrt{1-e^{2x}}} = \int \frac{e^{-x}dx}{\sqrt{e^{-2x}}}$, Put $u = e^{-x}$

$du = -e^{-x}dx$

$\int -\frac{du}{\sqrt{u^2-1}} = -\log\left[e^{-x} + \sqrt{e^{-2x}-1}\right] + C$

111. Ans: $\log \frac{x}{x+\cos x} + C$

Sol: $\int \frac{x+\cos x - x + x\sin x}{x(x+\cos x)}dx$

$\int \frac{x+\cos x}{x(x+\cos x)} - \frac{x(1-\sin x)}{x(x+\cos x)}dx$

$= \int\left[\frac{1}{x} - \left(\frac{1-\sin x}{x+\cos x}\right)\right]dx$

$= \log x - \log(x+\cos x) + C$

$= \log \frac{x}{x+\cos x} + C.$

112. Ans: $\int_0^{\frac{\pi}{6}} \frac{2x\,dx}{\tan x}.$

Sol: $\sin^{-1}\frac{x}{2} = t \Rightarrow \sin t = \frac{x}{2}$

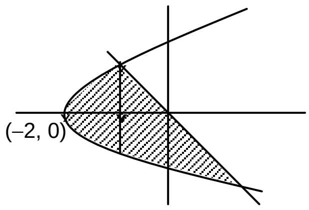
$\Rightarrow dx = 2\cos t\,dt$

$\Rightarrow \int_0^1 2\frac{\sin^{-1}\frac{x}{2}dx}{x} = \int_0^{\frac{\pi}{6}} 2t\cot t\,dt$

$= \int_0^{\frac{\pi}{6}} \frac{2t}{\tan t}dt$

113. Ans: $\frac{9}{2}$

Sol:



$2\int_{-2}^{-1}\sqrt{x+2}\,dx + \int_{-1}^{2}\left(-x+\sqrt{x+2}\right)dx$

$= \frac{4}{3}\left[(x+2)^{\frac{3}{2}}\right]_2^{-1} + \frac{-x^2}{2} + \frac{2}{3}\left[(x+2)^{\frac{3}{2}}\right]_{-1}^{2}$

$= \frac{9}{2}$ sq.units.

114. Ans: m + n.

Sol: $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^{2a} f(2a - xdx)$

= m + n

115. Ans: $\int_{-100}^{100} f(-x)dx$

Sol: $\int_{-100}^{100} f(-x)dx = \int_{100}^{-100} f(y)(-dy)$

$= \int_{-100}^{100} f(y)dy$

$= \int_{-100}^{100} f(-x)dx.$

116. Ans: 0

Sol: $\left(e^{x^3} + e^{-x^3}\right)\left(e^x - e^{-x}\right)$ is an odd function.

$$\therefore \int_{-1}^{1}\left(e^{x^3}+e^{-x^3}\right)\left(e^{x}-e^{-x}\right)dx = 0 .$$

117. Ans: y logy = tanx $\frac{dy}{dx}$

Sol: $y = e^{a\sin x}$

$\log y = a\sin x \Rightarrow \frac{1}{y}\frac{dy}{dx} = a\cos x$

$\Rightarrow \frac{dy}{dx} = \frac{\log y}{\sin x}\cos x$

$\Rightarrow y \log y = \frac{dy}{dx}\tan x .$

118. Ans: e^x

Sol: $\frac{dy}{dx} + \frac{1+x}{x}y = 1$

$\Rightarrow$ 1 F = $e^{\int\left(1+\frac{1}{x}\right)dx} = e^{x+\log x}$

$= x\,e^x.$

$\therefore$ I. F of given differential equation is e^x.

119. Ans: 3, 1

Sol: $\left(y - x\frac{dy}{dx}\right)^3 = a^2\left(\frac{dy}{dx}\right)^2 + b^2$

degree is 3 and order is 1.

120. Ans: $x + e^{-(x+y)} = C$

Sol: $x + y = z \Rightarrow \frac{dz}{dx} = 1 + \frac{dy}{dx}$

substituting,

$\Rightarrow e^{-z}dz = dx$

$\Rightarrow -e^{-z} = x + k$

$\Rightarrow -e^{-(x+y)} = x + k$

$\Rightarrow x + e^{-(x+y)} = C$