CET – MATHEMATICS – 2010

VERSION CODE: B - 2

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1. If A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $A^2 + xA + yI = 0$ for (x, y) = …………

a) (-4, 1) b) (-1, 3) c) (4, -1) d) (1, 3)

Ans (1)

Characteristic equation is $\lambda^2 - 4\lambda + 1 = 0$, using Cayley-Hamilton theorem

$A^2 - 4A + I = 0$ therefore x = -4 and y = 1

2. The constant term of the polynomial $\begin{vmatrix} x+3 & x & x+2 \\ x & x+1 & x-1 \\ x+2 & 2x & 3x+1 \end{vmatrix}$ is …………………

a) 0 b) 2 c) -1 d) 1

Ans (3)

Constant term is obtained putting x = 0

$\begin{vmatrix} 3 & 0 & 2 \\ 0 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix} = -1$

3. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are nonzero coplanar vectors, then $\left[2\vec{a} - \vec{b}\ \ 3\vec{b} - \vec{c}\ \ 4\vec{c} - \vec{a}\right]$ = …………

a) 25 b) 0 c) 27 d) 9

Ans (2)

Since three vectors are coplanar their scalar triple product is zero

$\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$ $(2\vec{a} - \vec{b}) \cdot [(3\vec{b} - \vec{c}) \times (4\vec{c} - \vec{a})]$

$= 12\vec{a} \cdot (\vec{b} \times \vec{c}) - 0 - 0 + 0 - \vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

4. A space vector makes the angles 150^0 and 60^0 with the positive direction of X- and Y-axes. The angle made by the vector with the positive direction of Z-axis is …………

a) 90^0 b) 60^0 c0 180^0 d) 120^0

Ans (1)

Let $\alpha = 150^\circ$ $\beta = 60^\circ$

we have $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1 \Rightarrow \cos^2\gamma = 0 \Rightarrow \gamma = 90^0$

5. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $3\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} =$ ……

a) -1 b) 1 c) -3 d) 3

Ans (3)

$\vec{a} + \vec{b} + \vec{c} = 0$ implies $\vec{a} + \vec{b} = -\vec{c}$ $\Rightarrow$ $(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = \vec{c}\cdot\vec{c}$

$\vec{a}.\vec{a} + \vec{b}\cdot\vec{b} + 2\vec{a}.\vec{b} = \vec{c}\cdot\vec{c}$ [$\vec{a}.\vec{a} = \vec{b}\cdot\vec{b} = \vec{c}\cdot\vec{c}$ = 1 because they are unit vectors]

implies $\vec{a}\cdot\vec{b} = -\frac{1}{2} = \vec{b}\cdot\vec{c} = \vec{a}\cdot\vec{c}$

Therefore given expression is -3

6. If a > b > 0, $\mathrm{Sec}^{-1}\left(\frac{a+b}{a-b}\right) = 2\mathrm{Sin}^{-1}x$, then x = …………..

a) $-\sqrt{\frac{b}{a+b}}$ b) $\sqrt{\frac{b}{a+b}}$ c) $-\sqrt{\frac{a}{a+b}}$ d) $\sqrt{\frac{a}{a+b}}$

Ans (2)

Given $\sec^{-1}\left(\frac{a+b}{a-b}\right) = 2\sin^{-1}x$ Let $\sec^{-1}\left(\frac{a+b}{a-b}\right) = \theta = \cos^{-1}\left(\frac{a-b}{a+b}\right)$

Then $x = \sin\frac{\theta}{2}$ $\cos\theta = \frac{a-b}{a+b}$ $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2} = \frac{b}{a+b}$ $\sin\frac{\theta}{2} = \sqrt{\frac{b}{a+b}}$

7. If $x \neq n\pi$, $x \neq (2n+1)\frac{\pi}{2}$, $n \in Z$, then $\frac{\mathrm{Sin}^{-1}(\mathrm{Cos}x) + \mathrm{Cos}^{-1}(\sin x)}{\mathrm{Tan}^{-1}(\mathrm{Cot}\,x) + \mathrm{Cot}^{2}(\mathrm{Tan}\,x)}$ = ………………

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

Wrong Options: you may identify it but taking a simple value for x like $x = \frac{\pi}{4}$

Most appropriate answer is '1' [but still given condition is not sufficient]

8. The general solution of $1 + \mathrm{Sin}^2 x = 3\,\mathrm{Sin}\,x\,.\,\mathrm{Cos}\,x$, $\mathrm{Tan}\,x \neq \frac{1}{2}$ is …………………..

a) $2n\pi + \frac{\pi}{4}$, $n \in Z$ b) $2n\pi - \frac{\pi}{4}$, $n \in Z$ c) $n\pi - \frac{\pi}{4}$, $n \in Z$ d) $n\pi + \frac{\pi}{4}$, $n \in Z$

Ans. (4)

$1 + \sin^2 x = 3\sin x\cos x$

divide by $\cos^2 x$ and rearrange we get

$2\tan^2 x - 3\tan x + 1 = 0$ implies $(2\tan x - 1)(\tan x - 1) = 0$ since $\tan x \neq ½$

Tan x = 1 its general solution is given by $n\pi + \frac{\pi}{4}$

9. The least positive integer n, for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is positive, is …………………

a) 3 b) 4 c) 1 d) 2

Ans (3)

$\frac{(1+i)^n}{(1-i)^{n-2}}$ = $(1 + i)^n(1 - i)^{2-n}$ given positive with n = 1

(1 + i) (1 - i) = 2

10. If x +iy = $\left(-1+i\sqrt{3}\right)^{2010}$, then x = ……………….

a) -2^{2010} b) 2^{2010} c) 1 d) -1

Ans (2)

$-1 + i\sqrt{3} = 2(\text{cis}\frac{2\pi}{3})$

therefore $(-1 + i\sqrt{3})^{2010} = 2^{2010}(\text{cis}\frac{2\pi}{3})^{2010} = 2^{2010}$ pure real itself is real part.

[observe that 2010 is multiple of 3 and $(\text{cis}\frac{2\pi}{3})^{2010} =1$]

11. (Sin θ + Cos θ) (Tan θ + Cot θ) = …………..

a) Sin θ . Cos θ b0 1 c) Sec θ + Cosec θ d) Sec θ Cosec θ

Ans (3)

$(\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right) = \left(\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}\right) = \sec\theta + \cos\theta$

12. The sides of a triangle are 6 + $\sqrt{12}$, $\sqrt{24}$. The tangent of the smallest angle of the triangle is …….

a) $\frac{1}{\sqrt{3}}$ b) $\sqrt{2}$ - 1 c) $\sqrt{3}$ d) 1

Ans (1)

Let a = 6 + $2\sqrt{3}$ b = $4\sqrt{3}$ c = $\sqrt{24}$ since c is smallest side C is smallest angle

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{\sqrt{3}}{2}$ ⇒ C = 30° therefore tan C = $1/\sqrt{3}$

13. A simple graph contains 24 edges. Degree of each vertex is 3. The number of vertices is ……………

a) 8 b) 12 c) 21 d) 16

Ans: (d)

Let n be the number of vertices. Degree of each vertex = 3

∴ Total degree = 3n. But total degree = 2 x number of edges. ⇒ Hence 3n = 2 (24) ⇒ n = 16

14. $\lim_{n\to\infty}\left\{n \operatorname{Sin}\frac{2\pi}{3n}.\operatorname{Cos}\frac{2\pi}{3n}\right\}$ = ...

a) $\frac{\pi}{6}$ b) $\frac{2\pi}{3}$ c) 1 d) $\frac{\pi}{3}$

Ans: (b)

$$\lim_{n\to\infty}\frac{1}{2}\ n\ .\ \sin\left(\frac{4\pi}{3n}\right) = \frac{1}{2}\lim_{n\to\infty}\left[\frac{\sin\left(\frac{4\pi}{3n}\right)}{\frac{4\pi}{3n}}\right]\frac{4\pi}{3n} = \frac{2\pi}{3}$$

15. The function f (x) = [x], where [x] denotes the greatest integer not greater than x, is……..

a) continuous for all nonintegral values of x.

b) continuous only at positive integral values of x.

c) continuous for all real values of x.

d) continuous only at rational values of x.

Ans (a)

The function f (x) = [x] in discontinuous at every integral value of x. If n is any integer, then $\lim_{x\to n^-}[x] = n - 1$ and $\lim_{x\to n^+}[x] = n$. $\therefore$ LHL $\neq$ RHL. f (x) = [x] is not continuous for rational or real values of x as both contain integers.

16. The greatest value of x satisfying 21 $\equiv$ 385 (mod x) and 587 $\equiv$ 167 (mod x) is …………….

a) 156 b) 32 c) 28 d) 56

Ans: (c)

$21 \equiv 385 \pmod{x} \Rightarrow x \mid -364$; $587 \equiv 167 \pmod{x} \Rightarrow x \mid 420$

$\therefore$ x = (364, 420) = 28

17. The number $(49^2 - 4)(49^3 - 49)$ is divisible by …………………..

a) 7! b) 9! c) 6! d) 5!

Ans: (d)

Now x = (364, 420) = 28

$(49^2 - 4)(49^3 - 49) = (49^2 - 2^2)(49^2 - 1)\ 49 = 51\ .\ 47\ .\ 50\ .\ 48\ .\ 49$,

which is the product of five consecutive integers, and hence divisible by 5!

18. The least positive integer x satisfying $2^{2010} \equiv 3x \pmod{5}$ is ………………..

a) 3 b) 4 c) 1 d) 2

Ans: (a)

$2^2 \equiv -1 \pmod{5} \Rightarrow 2^{2010} \equiv (-1)^{1005} \pmod{5} \Rightarrow 2^{2010} \equiv -1 \pmod{5} \Rightarrow -1 \equiv 2^{2010} \pmod{5}$

But $2^{2010} \equiv 3x \pmod{5}$. $\therefore -1 \equiv 3x \pmod{5} \Rightarrow x = 3$

19. If A and B are two square matrices of the same order such that AB = B ad BA = A then $A^2 + B^2$ is always equal to ……………….

a) I b) A + B c) 2 AB d) 2 BA

Ans: (b)

$A^2 + B^2 = A . A + B . B = A (BA) + B (AB) = (AB) A + (BA) B = BA + AB = A + B$

20. If A is a 3 x 3 nonsingular matrix and if |A| = 3, then $|(2A)^{-1}| =$

a) 24 b) 3 c) $\frac{1}{3}$ d) $\frac{1}{24}$

Ans: (d)

$|(2A)^{-1}| = \frac{1}{|2A|} = \frac{1}{2^3 .|A|} = \frac{1}{2^3 .3} = \frac{1}{24}$

21. If a, -a, b are the roots of $x^3 - 5x^2 - x + 5 = 0$, then b is a root of …………………..

a) $x^2 + 3x - 20 = 0$ b) $x^2 - 5x + 10 = 0$ c) $x^2 - 3x - 10 = 0$ d) $x^2 + 5x - 30 = 0$

Ans: (c)

$a + (-a) + b = 5 \Rightarrow b = 5$, which is a root of $x^2 - 3x - 10 = 0$

22. In the binomial expansion of $(1 + x)^{15}$, the coefficients of x^r and x^{r-3} are equal. Then r is ……….

a) 8 b) 7 c) 4 d) 6

Ans: (d)

$T_{r+1} = {}^{15}C_r\, x^r$. Co-efficient of $x^r = {}^{15}C_{r+3}$, $\therefore$ coefficient of $x^{r+3} = {}^{15}C_{r+3}$. Given, ${}^{15}C_r = {}^{15}C_{r+3}$

$\Rightarrow r + r + 3 = 15$

$\Rightarrow r = 6$

23. The n^{th} term of the series 1 + 3 + 7 + 13 + 21 + ……… Is 9901. The value of n is …………

a) 100 b) 90 c) 900 d) 99

Ans: (a)

By the method of differences, $t_n = 1 + (n - 1) n$

Given $1 + n (n - 1) = 9901 \Rightarrow n (n - 1) = 9900$ which is satisfied by n = 100

24. If $\frac{1}{(3 - 5x)(2 + 3x)} = \frac{A}{3 - 5x} = \frac{B}{2 + 3x}$, then A : B is ………………..

a) 2 : 3 b) 5 : 3 c) 3 : 5 d) 3 : 2

Ans: (b)

Here $A = \frac{5}{19}$, $B = \frac{3}{19}$ $\therefore$ A : B = 5 : 3

25. Which of the following is NOT true?

a) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$ is a tautology.

b) $\{(p \rightarrow q) \wedge (q \rightarrow r)\} \rightarrow (p \rightarrow r)$ is a tautology.

c) $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$.

d) $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$.

Ans: (a)

$\sim (p \rightarrow q) \equiv p \wedge \sim q$

$\therefore (p \wedge \sim q) \leftrightarrow (p \rightarrow q)$ is contradiction

26. If i, j, k are unit vectors along the positive direction of X, Y and Z-axes, then a FALSE statement in the following is ……………..

a) $\sum i \times (j + k) = \vec{0}$

b) $\sum i \times (j \times k) = \vec{0}$

c) $\sum i \,.\, (j \times k) = 0$

d) $\sum i \,.\, (j + k) = 0$

Ans: (c)

We have, $\Sigma i \,.\, (j \times k) = \Sigma\, i \,.\, i = \Sigma 1 = 3$

27. In P (X), the power set of a nonempty set X, an binary operation * is defined by

$A * B\ A \cup B\ \forall\ A, B \in P(X)$. Under *, a TRUE statement is ……………

a) identity law is not satisfied

b) inverse law is not satisfied

c) commutative law is not satisfied

d) associative law is not satisfied

Ans: (b)

Under the binary operation,

$A * B = A \cup B, \forall\ A, B \in P(x)$

Inverse of A doesn't exists because $A * B \neq \phi$, for any $B \in P(x)$

Where ϕ is the identity element in P (x)

28. The inverse of 2010 in the group Q^+ of all positive rational under the binary operation * defined by $a * b = \frac{ab}{2010}, \forall\ a, b \in Q^+$, is ………………

a) 2009

b) 2011

c) 1

d) 2010

Ans: (d)

By inspection, 2010 is the identity element

We know that inverse of identity element is itself

29. If the three functions f (x), g (x) and h (x) are such that h (x) = f (x) . g (x) and $f^{|}(x) \cdot g^{|}(x)$ = c, where c is a constant, then $\frac{f''(x)}{f(x)} = \frac{g''(x)}{g(x)} + \frac{2c}{f(x).g(x)}$ is equal to …………

a) $h^{|}(x) \cdot h^{||}(x)$ b) $\frac{h(x)}{h''(x)}$ c) $\frac{h''(x)}{h(x)}$ d) $\frac{h(x)}{h'(x)}$

Ans: (c)

Given that, h (x) = f (x) . g (x)

By differentiating twice, we get

h" (x) = f (x) g″ (x) + g (x) f″ (x) + 2 [f′ (x) . g′ (x)]

$\therefore \frac{f''(x)}{f(x)} + \frac{g''(x)}{g(x)} + \frac{2C}{f(x) \cdot g(x)}$

$= \frac{g(x).f''(x) + f(x).g''(x) + 2[f'(x).g'(x)]}{f(x).g(x)} = \frac{h''(x)}{h(x)}$

30. The derivative of e^{ax} Cos bx with respect to x si re^{ax} Cos $\left(bx + Tan^{-1}\frac{b}{a}\right)$. When a > 0, b > 0, the value of r is ……………..

a) $\sqrt{a^2 + b^2}$ b) $\frac{1}{\sqrt{ab}}$ c) ab d) a + b

Ans: (a)

y = e^{ax} . cos bx

$\therefore \frac{dy}{dx}$ = e^{ax} (-b sin bx) + cos bx (a . e^{ax})

= e^{ax} [a . cos bx – b . sin bx] = e^{ax} [r cos (bx + α)]

where $\alpha = \tan^{-1}\frac{b}{a}$ and r = $\sqrt{a^2 + b^2}$ = $re^{ax}\left[\cos\left(bx + \tan^{-1}\frac{b}{a}\right)\right]$

$\therefore$ r = $\sqrt{a^2 + b^2}$

31. The chord of the circle $x^2 + y^2 - 4x = 0$ which is bisected at (1, 0) is perpendicular to the line ……..

a) y = x b) x + y = 0 c) x = 1 d) y = 1

Ans: (d)

$x^2 + y^2 - 4x = 0$ $\therefore$ centre = (2, 0)

$\therefore$ slope of the line joining (1, 0) and (2, 0) = 0 = slope of the radius

$\therefore$ y = 1 is perpendicular to the chord, because it is parallel to radius.

32. In ΔABC, if a = 2, B = $\tan^{-1}\frac{1}{2}$ and C = $\tan^{-1}\frac{1}{3}$, then (A, b) =

a) $\left(\frac{3\pi}{4}, \frac{2}{\sqrt{5}}\right)$ b) $\left(\frac{\pi}{4}, \frac{2\sqrt{2}}{\sqrt{5}}\right)$ c) $\left(\frac{3\pi}{4}, \frac{2\sqrt{2}}{\sqrt{5}}\right)$ d) $\left(\frac{\pi}{4}, \frac{2}{\sqrt{5}}\right)$

Ans: (c)

$A = \pi - \left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

And $\tan B = \frac{1}{2} \Rightarrow \sin B = \frac{1}{\sqrt{5}}$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2}{\sin\frac{3\pi}{4}} = \frac{b}{\frac{1}{\sqrt{5}}} \Rightarrow b = \frac{2\sqrt{2}}{\sqrt{5}}$

33. The straight line 2x + 3y – k = 0, k > 0 cuts the X- and Y-axes at A and B. The area of ΔOAB, where O is the origin, is 12 sq. units. The equation of the circle having AB as diameter is ……………

a) $x^2 + y^2 - 6x - 4y = 0$ b) $x^2 + y^2 + 4x - 6y = 0$

c) $x^2 + y^2 - 6x + 4y = 0$ d) $x^2 + y^2 - 4x - 6y = 0$

Ans: (a)

x – and y – intercepts of 2x + 3y – k = 0 are $\frac{k}{2}$ and $\frac{k}{3}$

$\therefore$ area of the triangle = $\frac{1}{2}\left(\frac{k}{2}\right)\left(\frac{k}{3}\right) = 12 \Rightarrow k = 12$

and 2x + 3y – 12 = 0 is diameter to the circle $x^2 + y^2 - 6x - 4y = 0$

Because it passes through the center (3, 2)

34. Let P (x, y) be the midpoint of the line joining (1, 0) to a point on the curve $y^2 = \begin{vmatrix} x+1 & x+2 \\ x+3 & x+5 \end{vmatrix}$.

Then locus of P is symmetrical about …………..

a) Y-axis b) X-axis c) x = 1 d) y = 1

Ans: (b)

$y^2 = \begin{vmatrix} x+1 & x+2 \\ x+3 & x+5 \end{vmatrix} = \begin{vmatrix} x+1 & 1 \\ 2 & 1 \end{vmatrix} = x - 1$

$y^2 = x - 1$, which is a parabola

$\therefore$ parametric representation is $\left(\frac{t^2}{4} + 1, \frac{t}{2}\right)$

$\therefore$ Mid point of $\left(\frac{t^2}{4}+1, \frac{t}{2}\right)$ and (1, 0) is P (x, y) = $\left(\frac{t^2}{8}+1, \frac{t}{4}\right)$

which is similar to $(at^2 + h, 2at + k)$

$\therefore$ locus is a parabola which is symmetric about y = 0 (or) x – axis

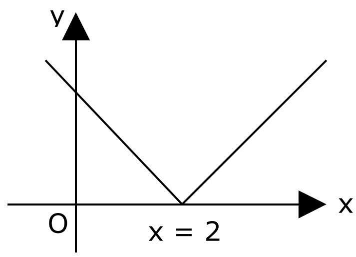
35. The function f (x) = |x – 2| + x is ……………….

a) differentiable at both x = 2 and x = 0. b) differentiable at x = 2 but not at x = 0.

c) continuous at x = 2 but not at x = 0. d) continuous at both x = 2 and x = 0.

Ans: (d)

f (x) = |x – 2| + x is continuous at x = 2 and x = 0



36. Let R be an equivalence relation defined on a set containing 6 elements. The minimum number of ordered pairs that R should contain is ……………….

a) 12 b) 6 c) 64 d) 36

Ans: (b)

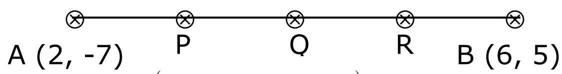
If A = {1, 2, 3, 4, 5, 6}

Then minimum number of ordered pairs in R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} are 6.

37. The line joining A (2, -7) and B (6, 5) is divided into 4 equal parts by the points P, Q and R such that AQ = RP = QB. The midpoint of PR is …………………..

a) (4, 12) b) (-8, 1) c) (4, -1) d) (8, -2)

Ans: (c)

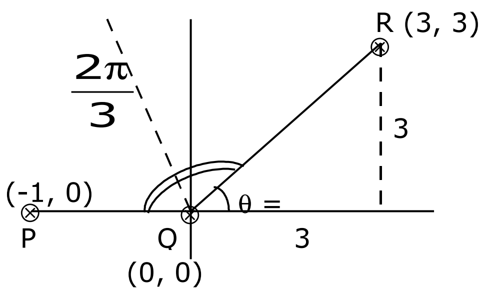


Mid point $\equiv$ Q $\equiv \left(\frac{6+2}{2}, \frac{-7+5}{2}\right) \equiv (4, -1)$

38. Let P $\equiv$ (-1, 0), Q $\equiv$ (0, 0) and R $\equiv$ $(3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is ……………

a) $x - \sqrt{3}y = 0$ b) $\sqrt{3}x - y = 0$ c) $x + \sqrt{3}y = 0$ d) $\sqrt{3}x + y = 0$

Ans: (d)



$\tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

angle $\left(\frac{\pi}{3}+\frac{\pi}{3}\right) = \frac{2\pi}{3}$

slope = tan $\frac{2\pi}{3}$ = $-\sqrt{3} \Rightarrow \sqrt{3}x + y = 0$

39. If m is the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$, then $(h + bm)^2$ = ……..

a) $(a + b)^2$ b) $(a - b)^2$ c) $h^2 + ab$ d) $h^2 - ab$

Ans: (b)

$m + m_2 = \frac{-2h}{b}$ $\quad m\, m_2 = \frac{a}{b}$

$m + \frac{a}{mb} = \frac{-2h}{b} \Rightarrow \frac{m^2b + a}{mb} = \frac{-2h}{b}$

$m^2 b + a = -2hm$

$m^2b + 2hm = -a \Rightarrow m^2b^2 + 2hmb = -ab$

$\Rightarrow m^2b^2 + 2hmb + h^2 = h^2 - ab \Rightarrow (h + bm)^2 = h^2 - ab$

40. $\text{Cot } 12^0 \text{ Cot } 102^0 + \text{Cot } 102^0 \text{ Cot } 66^0 + \text{Cot } 66^0 \text{ Cot } 12^0$ = ……………

a) -2 b) 1 c) -1 d) 2

Ans: (b)

$\cot(102^\circ) = -\tan 12^\circ$

$\therefore \cot 12^\circ\ (-\tan 12^\circ) + \cot 66^\circ\ [-\tan 12^\circ + \cot 12^\circ]$

$= -1 + \cot 66^\circ \left[\frac{1 - \tan^2 12^0}{\tan 12^0}\right]$

$= -1 + \cot 66^\circ \times \cot 24^\circ \times 2 = -1 + \cot 66^\circ (\tan 66^\circ) \times 2$

$= -1 + 2 = 1$

41. A wire of length 20 cm is bent in the form of a sector of a circle. The maximum area that can be enclosed by the wire is …………….

a) 20 sq. cm b) 25 sq. cm c) 10 sq. cm d) 30 sq. cm

Ans: (b)

$P = 2r + s$ $\quad A = \frac{1}{2}rs \,.\, \frac{1}{2}\, r \,.\, (20 - 2r)$

$2r + s = 20$ $\quad A = 10r - r^2$

$\frac{dA}{dr} = 10 - 2r$ $\quad \frac{dA}{dr} = 0 \Rightarrow r = 5$

$A_{max} = 50 - 25 = 25$

42. Two circles centered at (2, 3) and (5, 6) intersect each other. If the radii are equal, the equation of the common chord is …………….

a) $x + y + 1 = 0$ b) $x - y + 1 = 0$ c) $x + y - 8 = 0$ d) $x - y - 8 = 0$

Ans: (c)

Given circles are $(x - 2)^2 + (y - 3)^2 = r^2$ - (1)

$(x - 5)^2 + (y - 6)^2 = r^2$ - (2)

Radical axis is

(1) – (2) $-4x + 10x - 6y + 12y + 4 + 9 - 25 - 36 = 0$

$6x + 6y - 48 = 0$

$x + y - 8 = 0$

43. Equation of the circle centered at (4, 3) touching the circle $x^2 + y^2 = 1$ externally, is …………

a) $x^2 + y^2 - 8x - 6y + 9 = 0$ b) $x^2 + y^2 + 8x + 6y + 9 = 0$

c) $x^2 + y^2 + 8x - 6y + 9 = 0$ d) $x^2 + y^2 - 8x + 6y + 9 = 0$

Ans: (a)

By inspection

44. The points (1, 0), (0, 1), (0, 0) and (2k, 3k), $k \neq 0$ are concyclic if k = ………………..

a) $\frac{1}{5}$ b) $-\frac{1}{5}$ c) $-\frac{5}{13}$ d) $\frac{5}{13}$

Ans: (d)

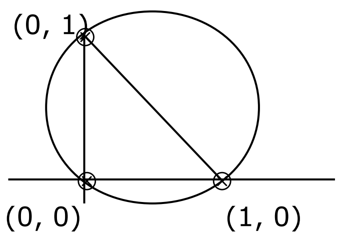
$x(x - 1) + y(y - 1) = 0$

$x^2 + y^2 - x - y = 0$

$4k^2 + 9k^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$13k = 5 \Rightarrow k = \frac{5}{13}$



45. The locus of the point of intersection of the tangents drawn at the ends of a focal chord of the parabola $x^2 = -8y$ is …………

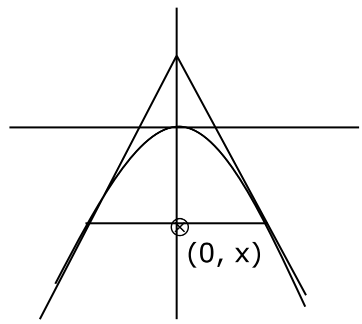
a) $x = 2$ b) $x = -2$ c) $y = 2$ d) $y = -2$

Ans: (c)

$x^2 = -8y$ $4a = -8 \Rightarrow a = -2$

$y = -\frac{x^2}{8}$

$y' = -\frac{x}{4}$



46. The condition for the line $y = mx + c$ to be a normal to the parabola $y^2 = 4ax$ is …………….

a) $c = -2am - am^3$ b) $c = -\frac{a}{m}$ c) $c = \frac{a}{m}$ d) $c = 2am + am^3$

Ans: (a)

$mx - y + c = 0$ - (1) $yy_1 = 2a(x + x_1)$

$2ax - yy_1 + 2ax_1 = 0$

47. The eccentric angle of the point $\left(2, \sqrt{3}\right)$ lying on $\frac{x^2}{16} + \frac{y^2}{4} = 1$ is ……………..

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

Ans: (c)

$x = 4\cos\theta$ $\cos\theta = \frac{1}{2}$

$y = 2\sin\theta$ $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$

48. The distance of the focus of $x^2 - y^2 = 4$, from the directrix which is nearer to it, is …………….

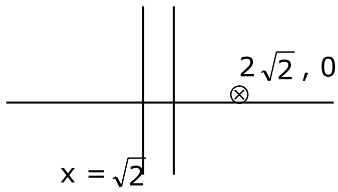
a) $4\sqrt{2}$ b) $8\sqrt{2}$ c) $2\sqrt{2}$ d) $\sqrt{2}$

Ans: (d)

$x^2 - y^2 = 2^2$

$(\pm ae, 0) = (\pm 2\sqrt{2}, 0)$

dir: $x = \pm \frac{a}{e} \Rightarrow x = \pm\sqrt{2}$



49. If $\int f(x) \text{ Sin } x. \text{ Cos } x \, dx = \frac{1}{2(b^2 - a^2)} \text{ Log } f(x) + c$, where c is the constant of integration, then $f(x) =$ …………………….

a) $\frac{2}{ab \text{ Cos } 2x}$ b) $\frac{2}{(b^2 - a^2)\text{ Cos } 2x}$ c) $\frac{2}{ab \text{ Sin } 2x}$ d) $\frac{2}{(b^2 - a^2)\text{ Sin } 2x}$

Ans: (b)

Method of inspection

$$\frac{d}{dx}\left[\frac{1}{2(b^2 - a^2)} \cdot \log\left[\frac{2}{(b^2 - a^2)\cos 2x}\right]\right]$$

$$= \frac{1}{2(b^2 - a^2)} \cdot \frac{d}{dx}\left[\log\left(\frac{2}{b^2 - a^2}\right) + \log\left[\frac{1}{\cos 2x}\right]\right]$$

$$= \frac{1}{2(b^2 - a^2)} \cdot \left[0 - \frac{d}{dx}(\log \cos 2x)\right] = \frac{1}{2(b^2 - a^2)}\left[\frac{\sin 2x \cdot 2}{\cos 2x}\right] = \frac{2}{b^2 - a^2} \cdot \frac{\sin x \cos x}{\cos 2x}$$

50. If $\int \frac{\sqrt{x}}{x(x+1)}\,dx = k\,\text{Tan}^{-1}m$, then (k, m) is ……………

a) (2, x) b) (1, x) c) $(1, \sqrt{x})$ d) $(2, \sqrt{x})$

Ans: (d)

$$\int \frac{\sqrt{x}}{x\,(x+1)}\,dx = k \tan^{-1}(m)$$

$$= \int \frac{\frac{1}{\sqrt{x}}}{x+1}\,dx = 2\int \frac{d(\sqrt{x})}{(\sqrt{x})^2+1} = 2\tan^{-1}(\sqrt{x}) \therefore k = 2,\ m = \sqrt{x}$$

51. $\int_0^{\frac{\pi}{4}} \frac{\text{Sin}\,x + \text{Cos}\,x}{3 + \text{Sin}\,2x}\,dx =$ ………………

a) $\frac{1}{4}$ Log 3 b) Log 3 c) $\frac{1}{2}$ Log 3 d) 2 Log 3

Ans: (a)

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{d[\sin x - \cos x]}{4 - (\sin x - \cos x)^2}$$

$$\sin x + \cos x = \frac{d}{dx}[\sin x - \cos x]$$

$$(\sin x - \cos x)^2 = 1 - \sin 2x$$

$$4 - (\sin x - \cos x)^2 = 3 + \sin 2x$$

$$= \frac{1}{4}\log\left[\frac{2 + (\sin x - \cos x)}{2 - (\sin x - \cos x)}\right]_0^{\frac{\pi}{4}} = \frac{1}{4}\log 3$$

52. $\int_0^1 x\,(1-x)^{3/2}\,dx =$ ……………

a) $\frac{-2}{35}$ b) $\frac{4}{35}$ c) $\frac{24}{35}$ d) $\frac{-8}{35}$]

Ans: (b)

$$\int_0^1 x\,(1-x)^{3/2}\,dx = \int_0^1 (1-x)\,x^{3/2}\,dx = \int_0^1 (x^{3/2} - x^{5/2})\,dx$$

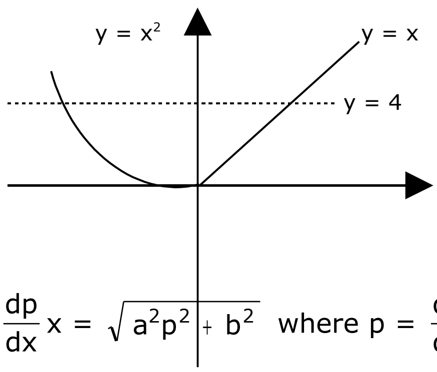
$$= \left[\frac{2}{5}x^{5/2} - \frac{2}{7}x^{7/2}\right]_0^1 = \frac{2}{5} - \frac{2}{7} = \frac{4}{35}$$

53. The area bounded by the curve $y = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$ and the lien y = 4 is ……………..

a) $\frac{32}{3}$ b) $\frac{8}{3}$ c) $\frac{40}{3}$ d) $\frac{16}{3}$

Ans: (c)

$A_2 = \frac{1}{2} \times 4 \times 4 = 8$



$A_1 = \int_0^4 \sqrt{y}\, dy = \frac{2}{3} y^{3/2}\Big|_0^4 = \frac{2}{3} (2^2)^{3/4} = \frac{16}{3}$

Area = $A_1 + A_2 = 8 + \frac{16}{3} = \frac{40}{3}$ sq. units

54. The order and degree of the differential equation $y = \frac{dp}{dx} x = \sqrt{a^2p^2 + b^2}$ where $p = \frac{dy}{dx}$ (here a and b are arbitrary constants) respectively are …………………

a) 2, 2 b) 1, 1 c) 1, 2 d) 2, 1

Ans: (a)

$y = \frac{d}{dx} \left[\frac{dy}{dx}\right] x + \sqrt{a^2 \left[\frac{dy}{dx}\right]^2} + b^2$

$y = x \frac{d^2y}{dx^2} + \sqrt{a^2\left[\frac{dy}{dx}\right]^2 + b^2}$

$\therefore \left(y - x . \frac{d^2y}{dx^2}\right)^2 = a^2 \left[\frac{dy}{dx}\right]^2 + b^2$

$\therefore$ order = 2 deg = 2

55. The general solution of the differential equation $2x \frac{dy}{dx} - y = 3$ is a family of ………………

a) hyperbolas b) parabolas c) straight lines d) circles

Ans: (b)

$2x\frac{dy}{dx} - y = 3$

$\frac{dy}{y - 3} = \frac{dx}{2x}$ $\therefore \log (y - 3) = \frac{1}{2} \log x + \log c$

$\log (y - 3) = \log (\sqrt{x}) + \log c$ $\therefore \log (y - 3) = \log (\sqrt{x}) + \log c$

$\therefore \frac{y - 3}{\sqrt{x}} = c$ $\therefore (y - 3)^2 = x c_1$ $\Rightarrow$ family of parabolas.

56. If x = a $\text{Cos}^3 \theta$ and y = a $\text{Sin}^3 \theta$, then $\frac{dy}{dx}$ = ………………

a) $\sqrt[3]{\frac{y}{x}}$ b) $\sqrt[3]{\frac{x}{y}}$ c) $-\sqrt[3]{\frac{x}{y}}$ d) $-\sqrt[3]{\frac{y}{x}}$

Ans: (d)

$x = a\cos^3\theta$ $y = a\sin^3\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\tan\theta$$

$\tan^3\theta = \frac{y}{x}$ $\therefore -\tan\theta = -\sqrt[3]{\frac{y}{x}}$

57. If y = $\text{Tan}^{-1}\sqrt{x^2-1}$, then the ratio $\frac{d^2y}{dx^2} : \frac{dy}{dx}$ = …………………

a) $\frac{x(x^2-1)}{1+2x^2}$ b) $\frac{1-2x^2}{x(x^2-1)}$ c) $\frac{1+2x^2}{x(x^2+1)}$ d) $\frac{x(x^2+1)}{1-2x^2}$

Ans: (b)

$y = \tan^{-1}\sqrt{x^2-1}$

$$y' = \frac{1}{1+(x^2-1)} \cdot \frac{1}{2\sqrt{x^2-1}} \cdot 2x = \frac{1}{x\sqrt{x^2-1}}$$

$$y'' = \frac{-1}{(x\sqrt{x^2-1})^2}\left[x \cdot \frac{1}{x\sqrt{x^2-1}} \cdot 2x + \sqrt{x^2-1}\right]$$

$$= \frac{-1}{x^2(x^2-1)} \times \frac{(x^2+x^2-1)}{\sqrt{x^2-1}} = \frac{1-2x^2}{x^2(x^2-1)\sqrt{x^2-1}}$$

$$\therefore \frac{y''}{y} = \frac{1-2x^2}{x^2(x^2-1)\sqrt{x^2-1}} \times x\sqrt{x^2-1} = \frac{1-2x^2}{x(x^2-1)}$$

58. P is the point of contact of the tangent from the origin to the curve y = $\text{Log}_e x$. The length of the perpendicular drawn from the origin to the normal at P is ……………

a) $\frac{1}{2e}$ b) $\frac{1}{e}$ c) $2\sqrt{e^2+1}$ d) $\sqrt{e^2+1}$

Ans: (d)

$y = \log_e x$ $\frac{dy}{du} = \frac{1}{x}$

Equation of tangent $y - y_1 = \frac{1}{x_1}(x - x_1)$

passing through (0, 0), $-y_1 = \frac{1}{x_1}(-x_1) \Rightarrow y_1 = 1, \therefore x_1 = e$

$\therefore$ point is (e, 1)

Equation of normal at (e, 1) is $y - 1 = -e(x - e)$

$\Rightarrow ex + y - 1 - e^2 = e$

length of perpendicular = $\left|\frac{-c}{\sqrt{a^2+b^2}}\right| = \frac{e^2+1}{\sqrt{e^2+1}} = \sqrt{e^2+1}$

59. For the curve $4x^5 = 5y^4$, the ratio of the cube of the subtangent at a point on the curve to the square of the subnormal at the same point is ………………….

a) $x\left(\frac{4}{5}\right)^4$ b) $y\left(\frac{5}{4}\right)^4$ c) $\left(\frac{4}{5}\right)^4$ d) $\left(\frac{5}{4}\right)^4$

Ans: (c)

$4x^5 = 5y^4 \Rightarrow 20x^4 = 20y^3 \cdot y'$

$\Rightarrow y' = \frac{x^4}{y^3}$

$$\therefore \frac{(SN)^3}{(ST)^2} = \frac{\left(y^4/x^4\right)^3}{\left(\frac{x^4}{y^2}\right)^2} = \frac{y^{12}}{x^{12}} \times \frac{y^4}{x^8} = \frac{y^{16}}{x^{20}} = \left(\frac{4}{5}\right)^4$$

60. The set of real values of x for which $f(x) = \frac{x}{\operatorname{Log} x}$ is increasing, is ……………..

a) $\{x : x \geq e\}$ b) empty c) $\{x : x < e\}$ d) $\{1\}$

Ans: (a)

$f(x) = \frac{x}{\log x}$

$f'(x) = \frac{\log x . 1 - x . \frac{1}{x}}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$

f (x) is increasing $\Rightarrow f'(x) > 0$

$$\Rightarrow \frac{\log x - 1}{(\log x)^2} > 0 \Rightarrow \log x - 1 > 0 \Rightarrow x > e$$